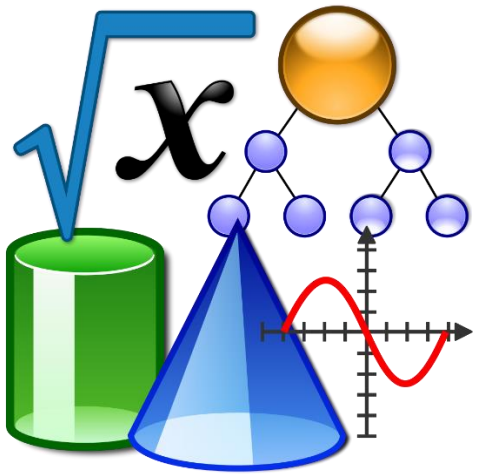
NPS Learning in Place

Geometry



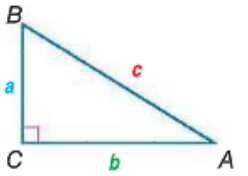
Name:_____ School:_____ Teacher: _____

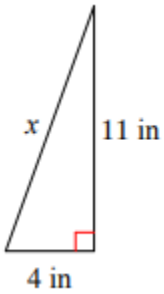
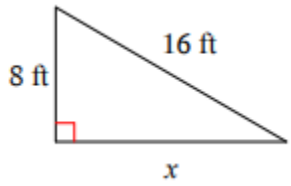
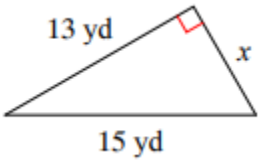
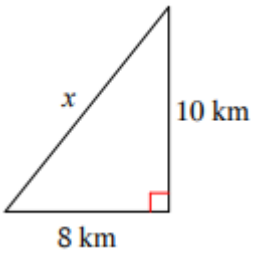
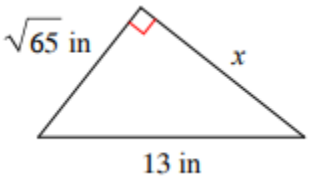
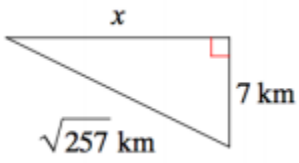
May 18 - June 5

Week 1	• **Right Triangles**
Week 2	• **Quadrilaterals**
Week 3	• **Polygons**

Week 1

Day 1: Right Triangles and Trigonometry

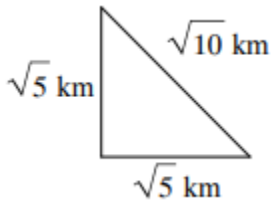
Using the Pythagorean Theorem and inequalities	
Remember that for a right triangle with legs *a* and *b*, and hypotenuse *c* : $a^2 + b^2 = c^2$ It's also true that if three sides of a triangle satisfy the relationship $a^2 + b^2 = c^2$, then those three sides form a *right* triangle.  	
Will three sides form a triangle? Triangle inequality theorem: Sum of two shortest sides > Third side $a + b > c$	What kind of triangle will the three sides make? $a^2 + b^2 = c^2$ Right triangle $a^2 + b^2 > c^2$ Acute triangle $a^2 + b^2 < c^2$ Obtuse triangle

Pythagorean Theorem practice	
Find the missing side:	
1.  	2. 
3.  	4. 
5.  	6. 

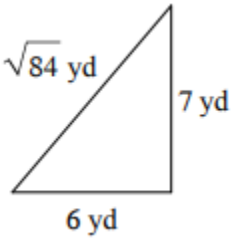
Inverse of the Pythagorean Theorem

Determine if the triangle is a right triangle. If not, determine if it is acute or obtuse:

1.



2.



Determine whether each set of numbers can be the sides of a triangle. If so, classify the triangle as right, acute, or obtuse:

3. 11, 60, 61

4. 6.2, 13.8, 20

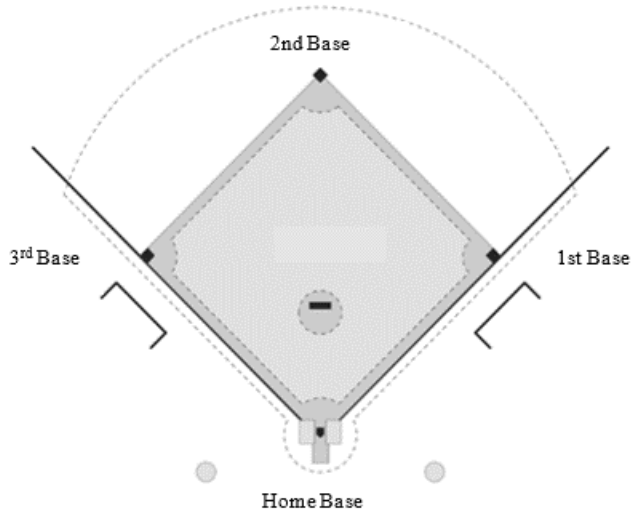
5. 15, 36, 39

6. 16, 18, 26

Take Me Out to the Ball Game

The four bases of a major league baseball field form a square which is 90 feet on each side. The pitcher stands on a pitching mound inside the square.

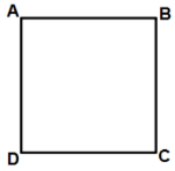
- The pitching mound is collinear to home plate and second base.
- The pitching mound is not equidistant from each base.
- The pitching mound is 60.5 feet from home plate.



To which base is the pitcher closest? Mathematically justify your answer.

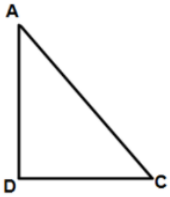
Day 2: Special Right Triangles

Perimeter of square ABCD = 4 cm



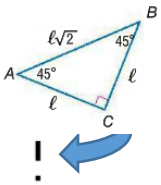
1.) Use the Pythagorean Theorem to solve for AC.

2.) Given the diagram below, fill in each angle measure and side value based on your work above and what you know about squares.



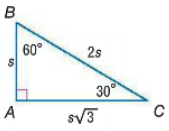
In a 45°-45°-90° triangle, the legs ℓ are congruent and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg.

Symbols In a 45°-45°-90° triangle, $\ell = \ell$ and $h = \ell\sqrt{2}$.

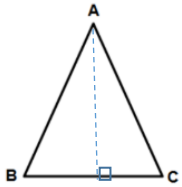


In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s, and the length of the longer leg ℓ is $\sqrt{3}$ times the length of the shorter leg.

Symbols In a 30°-60°-90° triangle, $h = 2s$ and $\ell = s\sqrt{3}$.



Perimeter of equilateral ΔABC = 6 cm

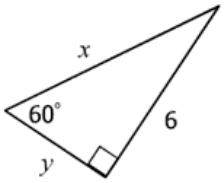


1.) Use the Pythagorean Theorem to solve for the altitude.

2.) Given the diagram below, fill in each angle measure and side value based on your work above and what you know about equilateral triangles.

Use the properties of special triangles to find the values of the variables in the problems below:

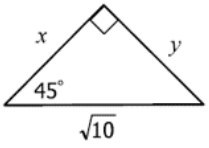
1.



$x =$ _____

$y =$ _____

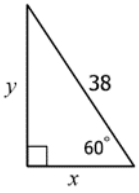
2.



$x =$ _____

$y =$ _____

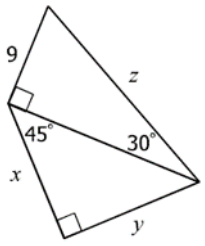
3.



$x =$ _____

$y =$ _____

4.

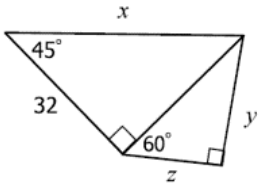


$x =$ _____

$y =$ _____

$z =$ _____

5.



$x =$ _____

$y =$ _____

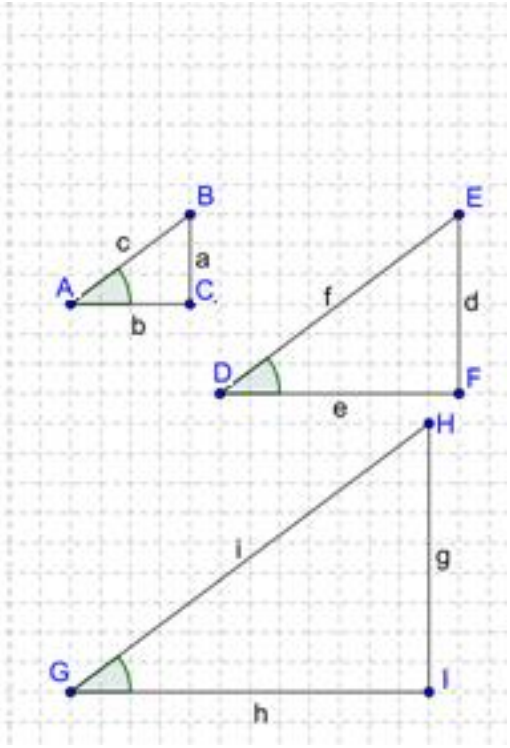
$z =$ _____

Day 3: Right Triangle Trigonometry

Vocabulary

Reference Angle: the marked angle
Hypotenuse (HYP): the side opposite the right angle in a right triangle
Opposite (OPP): the side opposite the reference angle
Adjacent (ADJ): the side adjacent to (next to) the reference angle

Use the diagram of three right triangles, ΔABC, ΔDEF, and ΔGHI, to complete the tables and answer the following questions. Remember to reduce fractions!



1. Count, and use the Pythagorean Theorem to complete the table.

	ΔABC	ΔDEF	ΔGHI	ΔJKL
OPP	a =	d =	g =	j =
ADJ	b =	e =	h =	k =
HYP	c =	f =	i =	l =

2. What do you notice about the lengths of the sides of the triangles?

3. What can you say about the triangles? (Hint: They are not congruent, but…) Why do you know this?

4. What is the scale factor of ΔDEF to ΔABC? ____ ΔGHI to ΔABC? ____

5. What do you know about $\angle C$, $\angle F$, and $\angle I$? What about $\angle A$, $\angle D$, and $\angle G$? Why do you know this?

6. Complete the table below. (The definitions of these terms can be found in the vocabulary section on the next page)

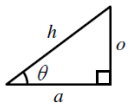
	Write the names of the Trig ratios:			_____	_____	_____
	OPP	ADJ	HYP	$\frac{\text{OPP}}{\text{HYP}}$	$\frac{\text{ADJ}}{\text{HYP}}$	$\frac{\text{OPP}}{\text{ADJ}}$
ΔABC						
ΔDEF						
ΔGHI						

7. Make sure the ratios in the last three columns have been reduced. Do you notice any patterns?

Day 4 Notes: Using Trigonometry to Find Missing Values

Vocabulary

- Trigonometric Ratio: a ratio of the lengths of sides of a right triangle
- Sine (of the reference angle): the ratio $\frac{\text{OPP}}{\text{HYP}}$
- Cosine (of the reference angle): the ratio $\frac{\text{ADJ}}{\text{HYP}}$
- Tangent (of the reference angle): the ratio $\frac{\text{OPP}}{\text{ADJ}}$

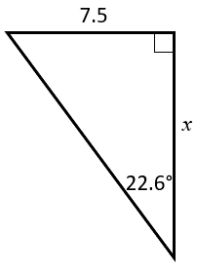


$\text{Sin}\,\theta = \frac{o}{h}$

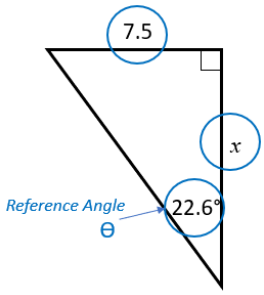
$\text{Cos}\,\theta = \frac{a}{h}$

$\text{Tan}\,\theta = \frac{o}{a}$

Which ratio should I use? (and another way to label the triangle)



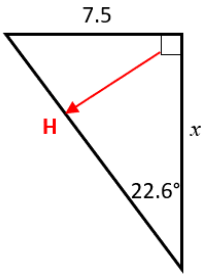
1. Look at a picture of the problem. You might have to draw a picture based on a description. You should have a right triangle with one right angle.



2. You should have at least *two* numbers or values that are given to you (here, 7.5 and 22.6°) and at least *one* number that you need to find (*x*). Circle them.

(If all the values in your problem are *sides*, you can use the Pythagorean theorem to find the missing value.)

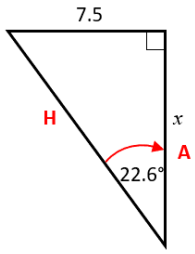
Here, you have two sides and one *angle*. We call that the "reference angle", Θ ("theta"). The reference angle is always one of the acute angles. Since we have a mix of sides and angles in our problem, we need to use one of the trigonometric ratios.



3. To figure out which ratio to use, we need to label our sides based on the reference angle. Start with the hypotenuse (H). It's the side opposite the right angle.

Think of it as the side that the right angle symbol points to.

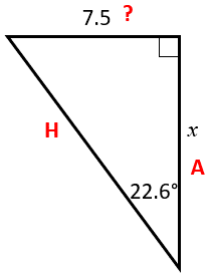
Label side H (the hypotenuse).



4. Once you identify the hypotenuse, draw an **A**rrow **A**cross the **A**cute (reference) **A**ngle.

See all the "**A**" words? That arrow points to the side **a**djacent to the reference angle.

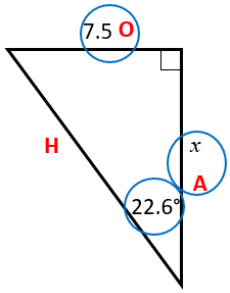
Label side **A** (the adjacent side).



5. Now what about the **O**ther side? It's on the **O**ther side of the triangle from the reference angle, or **O**pposite the reference angle.

That's the **o**pposite side.

Label side **O** (the opposite side).



6. Now look at which sides you circled. Use the ratio that includes those two sides.

Here, we circled the sides we labeled A and O, so we need the ratio that includes A and O.

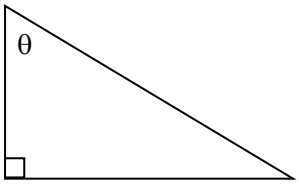
We'll use the tangent ratio: $\text{Tan}\,\theta = \frac{o}{a}$

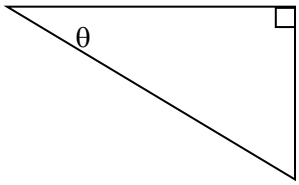
So we can write $\tan(22.6) = \frac{7.5}{x}$ and solve for *x* .

Day 4 Practice: Trigonometry Practice

MAKE SURE YOUR CALCULATOR IS SET TO "DEGREES"!!

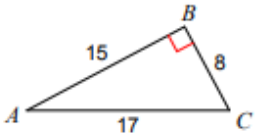
1. From the reference angle, θ, label the opposite side (OPP), adjacent side (ADJ), and hypotenuse (HYP) for each right triangle. (Label on the triangle)





Write each ratio for the triangle shown.

2.

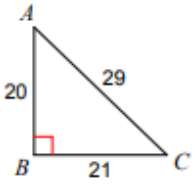


cos A = _____ cos C = _____

sin A = _____ sin C = _____

tan A = _____ tan C = _____

3.



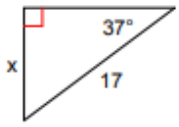
cos A = _____ cos C = _____

sin A = _____ sin C = _____

tan A = _____ tan C = _____

Find the length of the missing side.

4.



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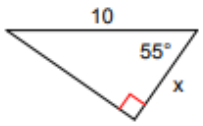

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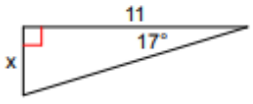

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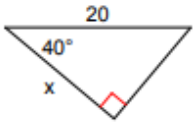

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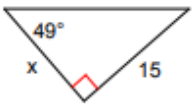

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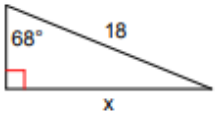
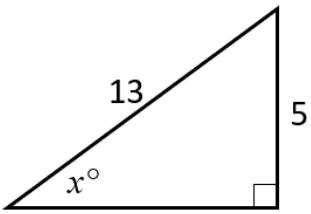


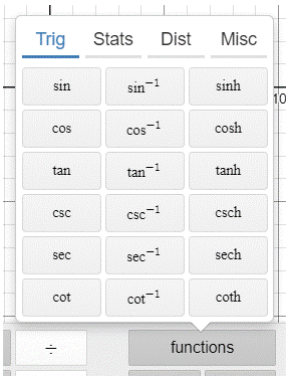
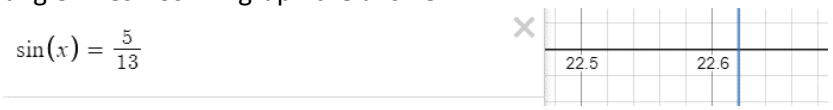
Finding the value of an Angle



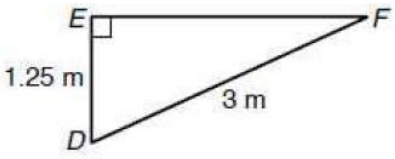
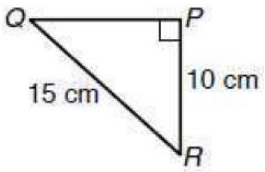
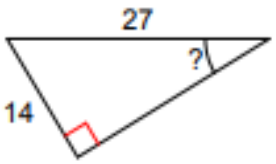
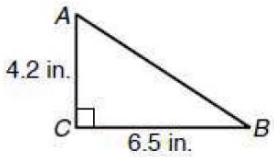
If we know the ratio of two sides in a right triangle, we can use that ratio to find the measure of the angles by using the inverse of the trigonometric functions (we say "sine inverse", etc.)

Inverse Trigonometric Functions
If $\sin A = \frac{O}{H}$, then $\sin^{-1}(\frac{O}{H}) = m\angle A$
If $\cos A = \frac{A}{H}$, then $\cos^{-1}(\frac{A}{H}) = m\angle A$
If $\tan A = \frac{O}{A}$, then $\tan^{-1}(\frac{O}{A}) = m\angle A$

(Make sure to set your calculator to DEGREES!)

Graphing Calculator	Desmos
The inverse trigonometry functions are usually the 2nd function of sin, cos, and tan: sin⁻¹ E cos⁻¹ F tan⁻¹ G sin cos tan It will look something like this: sin⁻¹(5/13) 22.61986495	You can find the inverse functions on the functions tab:  And you can type in: 
	OR
	You can just set up the trigonometric function with an unknown angle. Desmos will graph the answer. 

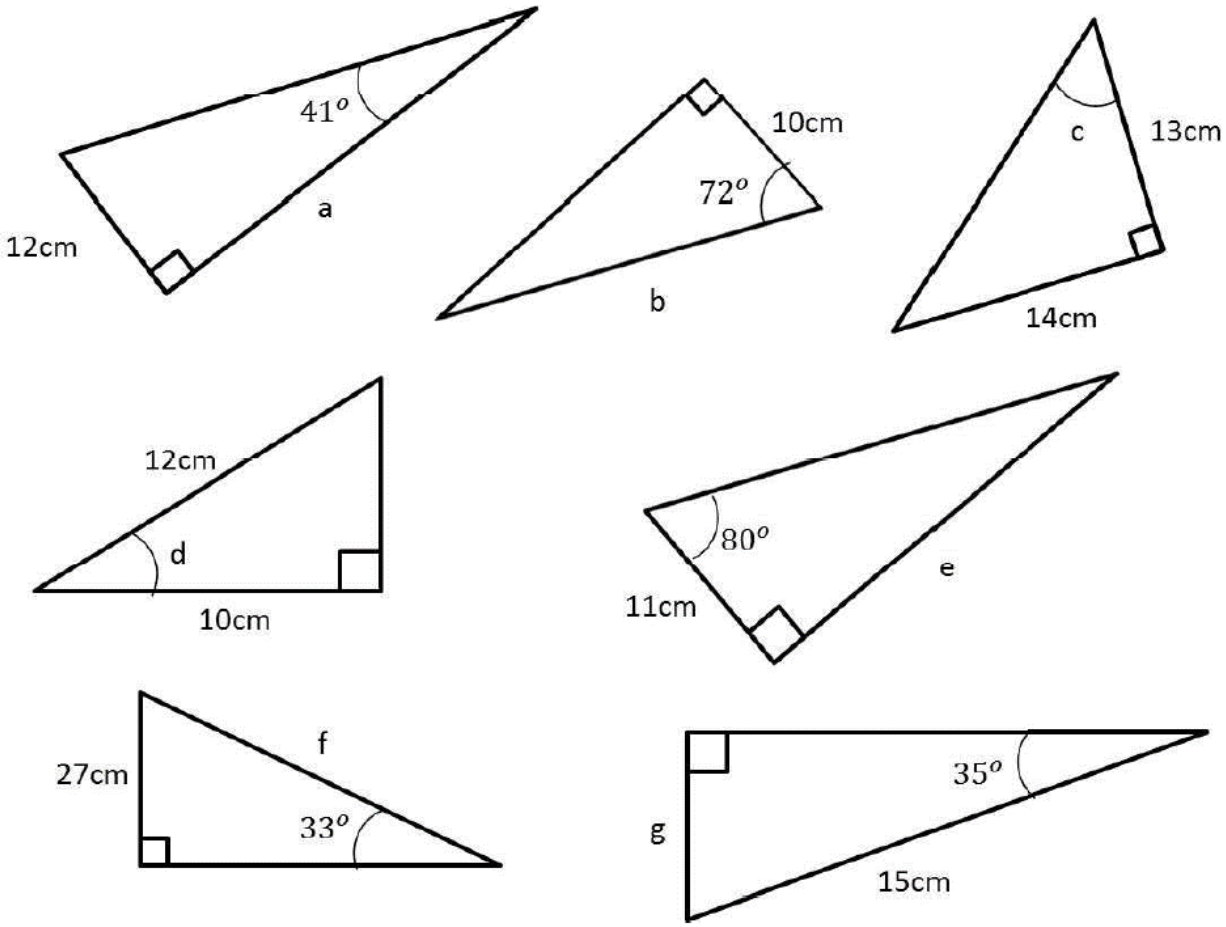
Practice:

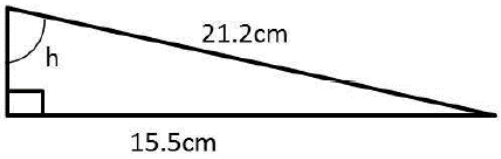
1. Find m$\angle F$  	2. Find m$\angle R$ 
3. Find the measure of the indicated angle:  	4. Find the measure of $\angle B$: 

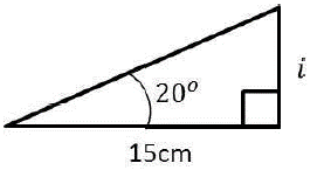
Day 5: Mixed Trigonometry Practice

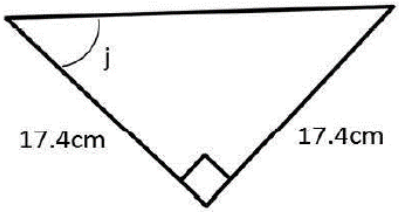
Will	Sin	Always	You	An	90	It's
32.4	12.5	11.5	47.1	62.4	5.5	37.8
Acute	Cos	A	Why	Argument	Degree	Because
45.6	51.4	47.0	13.8	49.6	45.0	27.4
Lose	Beach	Angle	Tan	Right	Obtuse	With
33.6	36.3	59.9	29.8	69.3	52.9	8.6

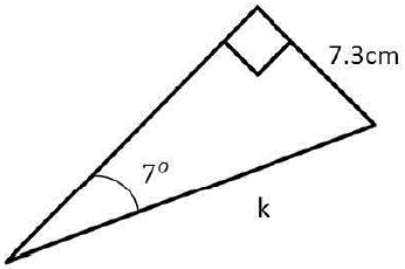
Find the missing side or angle labeled (round to the nearest tenth). Use the code above to translate your answer into part of the coded joke.

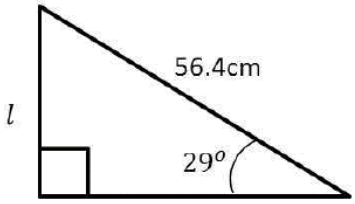


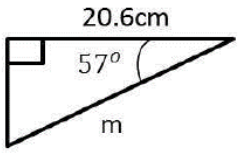


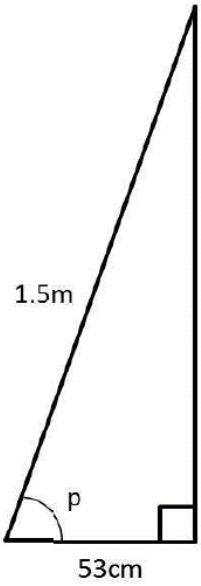


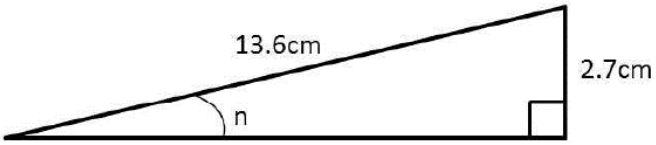












_____ _____ _____ _____ _____ _____ _____

a **b** **c** **d** **e** **f** **g**

_____ _____ _____ _____ **?**

h **i** **j** **k**

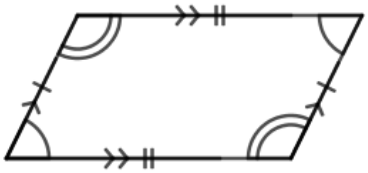
_____ _____ _____ _____

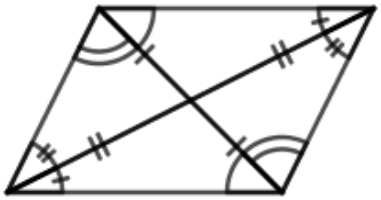
l **m** **n** **p**

Journal/ Writing Prompt: You are given a right triangle *ABC* where $\angle C$ is the right angle. You are told the lengths of sides *a* and *c* and the measure of $\angle B$. Explain how you would find the length of side *b* and the measure of $\angle A$.

Week 2 Day 1- Parallelograms

Parallelograms





Properties:

1) Opposite sides are parallel

2) Opposites sides are congruent

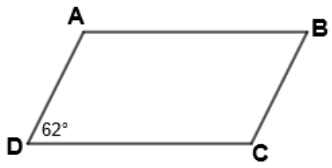
3) Opposite angles are congruent

4) Consecutive Interior (Same Side Interior) angles are supplementary.

5) Diagonals bisect each other

*Diagonals create alternate interior angles that are congruent.

Ex 1) Find the measure of the missing angles given Parallelogram ABCD.

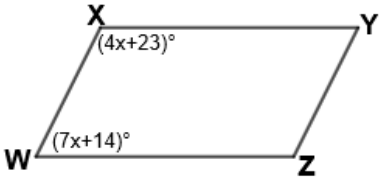


$m\angle A = 118° \rightarrow \quad \angle D$ and $\angle A$ are consecutive interior angles therefore $180 - 62 = 118$

$m\angle B = 62° \rightarrow \angle D$ and $\angle B$ are opposite angles therefore they congruent.

$m\angle C = 118° \rightarrow \angle D$ and $\angle C$ are consecutive interior angles therefore $180 - 62 = 118$

Ex 2) Given Parallelogram WXYZ, Find the $m\angle XYZ$.



Work: $\angle W$ and $\angle X$ are consecutive interior angles, therefore, the two angles are supplementary.

$4x + 23 + 7x + 14 = 180$

$11x + 37 = 180 \rightarrow$ Combine like terms

$11x = 143 \rightarrow$ Subtract 37 from each side

$x = 13 \rightarrow$ Divide each side by 11

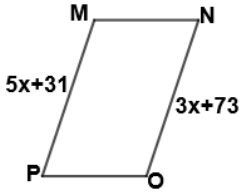
*Plug into $\angle W$

$7(13) + 14$

$m\angle W = 105°$

Answer: $m\angle XYZ = 105° \rightarrow \angle W \cong \angle XYZ$

Ex 3) Given Parallelogram MNOP, find the length of MP.



Work:

$5x + 31 = 3x + 73 \rightarrow$ Opposite sides are congruent

$2x + 31 = 73 \rightarrow$ Subtract $3x$ from each side

$2x = 42 \rightarrow$ Subtract 31 from each side

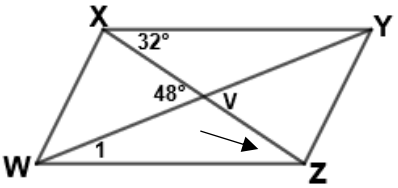
$x = 21 \rightarrow$ Divide each side by 2

*Substitute into MP

$5(21) + 31 = 136$

Answer: MP=136

Ex 4) Given Parallelogram WXYZ, find the measures of angle 1.



Work:

$m\angle XZW = 32° \rightarrow$ Alternate Interior Angles are congruent.

$m\angle MVZ = 132° \rightarrow$ Linear Pair with $\angle XVW$

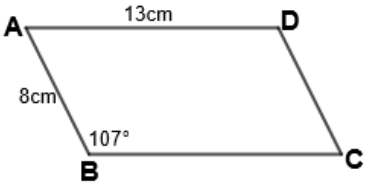
$32 + 132 = 164$

$180 - 164 = 16 \rightarrow$ Sum of the interior angles of a triangle equal 180°

Answer: $m\angle 1 = 16°$

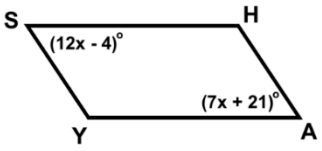
Directions: Each of the following are parallelograms. Use your knowledge of the properties of parallelograms to solve each of the following.

1) Find the missing angles and sides for the following parallelogram:

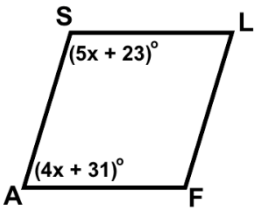


$m\angle A$=_____
$m\angle C$=_____
$m\angle D$ =_____
DC =_____
BC =_____

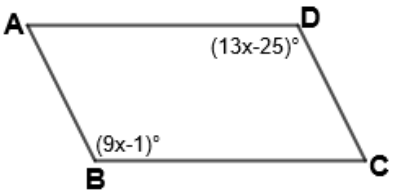
2) Find the $m\angle SYA$.



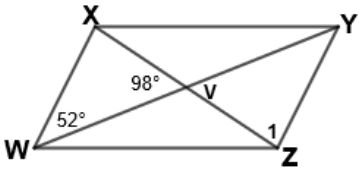
3) What is the $m\angle SLF$?



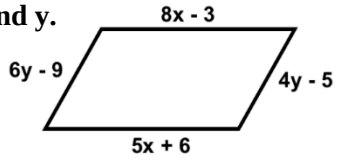
4) What is the $m\angle D$?



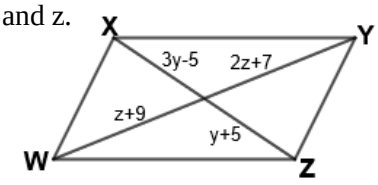
5) What is the measure of ∠1?



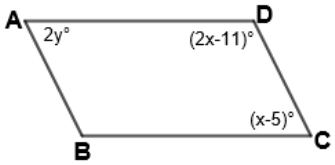
6) **Find the values of x and y.**



7) Find the values of y and z.



8) Sarah and David were given the following parallelogram. They were asked to solve for x and y.
Sarah set up her first step of the problem like $2x - 11 = x - 5$. David set up his problem like $2x - 11 + x - 5 = 180$. Who set up their problem correctly and explain why.



Day 2- Rectangles

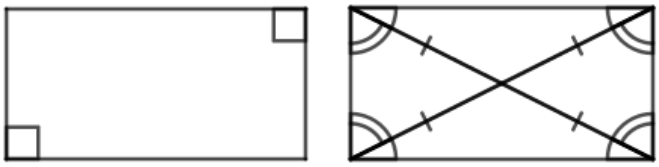
Rectangles

Properties: Has *ALL* parallelogram (see Day 1) properties PLUS:

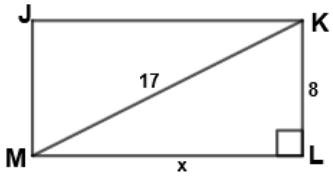
1) Four right angles

2) Diagonals are congruent.

*Diagonals create isoscles triangles



Ex 1) Given Rectangle JKLM where KM=17 and KL=8. What is the length of ML and the perimeter?



Work:

$a^2 + b^2 = c^2$

$x^2 + 8^2 = 17^2$ → Substitution

$x^2 + 64 = 289$ → Simplify

$x^2 = 225$ → Subtract 64 from both sides

$\sqrt{x^2} = \sqrt{225}$ → Square root both sides

$x = 15$

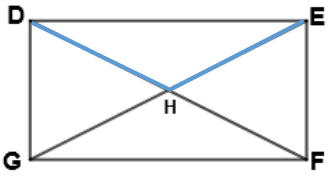
Answer: $ML = 15$

Perimeter: $8 + 8 + 15 + 15 = 46$

Ex 2) Given Rectangle DEFG. If $EH = 2x + 6$ and $DH = 6x - 10$. What is the length of DH and DF?

Work:

*Diagonals are congruent which means EH=DH.



$6x - 10 = 2x + 6$

$4x - 10 = 6$ → Subtract

$4x = 16$ → Add 10 to each side

$x = 4$ → Divide both sides by 4

*Substitute:

DH= $6(4) - 10$

DH=14 → Simplify

DF= EH+HG

EH=HG

DF=14+14

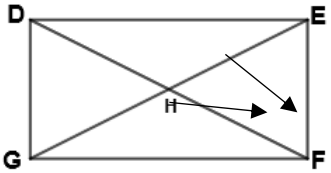
DF=28

Answer: $DH = 14$ & $DF = 28$

Ex 3) Given the $m\angle DFG = (8x - 24)°$ and $m\angle DFE = (2x + 2)°$. What is the value of $m\angle DFG$?

Work:

*Rectangles have right angles



$8x - 24 + 2x + 4 = 90$

$10x - 20 = 90$ → Combine like terms

$10x = 110$ → Add 20 to both sides

$x = 11$ → Divide each side by 10

*Substitute:

$8(11) - 24 = 64$

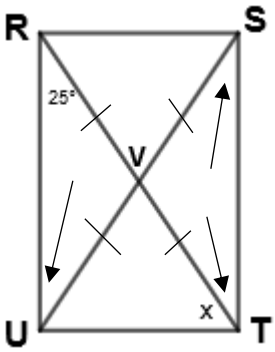
Answer: $m\angle DFG = 64°$

Ex 4) Given the $m\angle URV = 25°$, find the value of x.

Work:

*Diagonals create isosceles triangles.

So we can conclude that the $m\angle RUV, m\angle VST$ and $m\angle STV$ are all 25°.



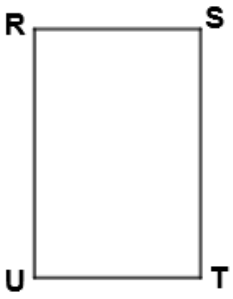
Rectangles also have 4 right angles so

$m\angle STV + m\angle VTU = 90°$

$90 - 25 = 65$

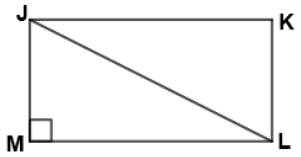
Answer: $x = 65°$

You try: Given Rectangle RSTU with RS= 7 and ST=15. Find RT.



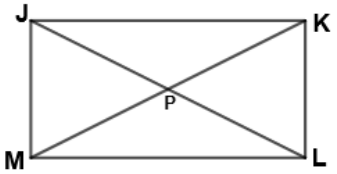
Directions: Each of the following are rectangles. Use your knowledge of the properties of rectangles to solve each of the following.

1) Given $JL = 10$ and $JM = 8$. What is the length of JK and the perimeter of Rectangle JKLM?

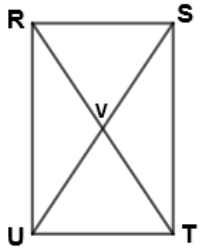


2) If DG=10 and DE=24, what is the length of DF?

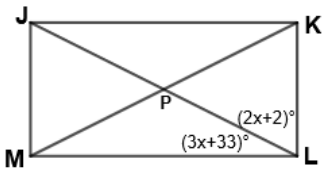
3) If $JL = 18x - 2$ and $KM = 9x + 25$, what is the value of x?



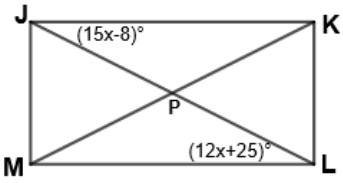
4) Given $RV = 9x - 31$ and $UV = 5x + 13$. What is the value of x and the length of US?



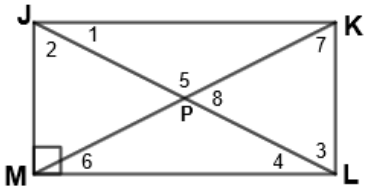
5) What is the value of x?



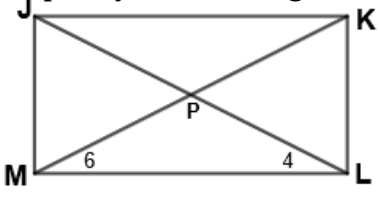
6) Find x.



7) Find each measure if the $m\angle 2 = 20°$.



8) Given the following quadrilateral. If the $m\angle 6 = 35°$ and $m\angle 4 = 45°$, could quadrilateral JKLM be a rectangle? Explain your reasoning.



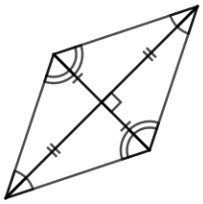
Day 3- Rhombi & Squares

Rhombus:

Properties: Has *ALL* parallelogram (see Day 1) properties PLUS:

1) All sides are congruent
2) Diagonals are perpendicular
3) Diagonals bisect opposite angles

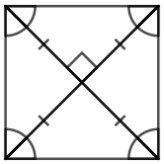
*Hint: Diagonals create right angles



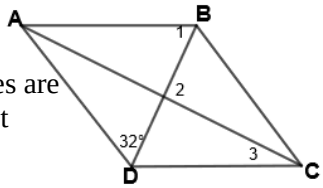
Square:

Properties: Has *ALL* parallelogram (see Day 1) properties PLUS:

1) All sides are congruent
2) Diagonals are congruent
3) Diagonals are perpendicular
4) Diagonals bisect opposite angles



Ex 1) Given Rhombus ABCD, find the missing angles.



Work:

$m\angle 1 = 32°$ → Opposite angles are congruent and diagonals bisect opposite angles.

$m\angle 2 = 90°$ →Diagonals are perpendicular.

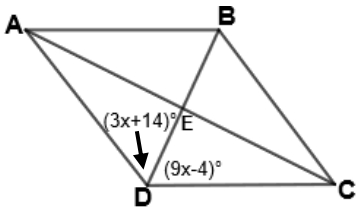
$m\angle 3 = 58°$ → $m\angle BDC = 32°$ because diagonals bisect opposite angles. At the intersection of the diagonals is a right angle because the diagonals are perpendicular.

Therefore, the sum of the angles of triangle add up to 180°.

Ex 2) Given Rhombus ABCD, find the value of x.

Work:

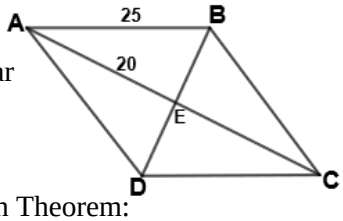
*Diagonals bisect opposite angles.



$9x - 4 = 3x + 14$
$6x - 4 = 14$ → Subtract $3x$ from both sides
$6x = 18$ → Add 4 to both sides
$x = 3$ → Divide both sides by 6

Answer: $x = 3$

Ex 3) Find BE.



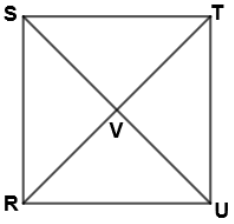
Work:

*Diagonals are perpendicular and create right triangles.

To find BE, use Pythagorean Theorem:
$a^2 + b^2 = c^2$
$x^2 + 20^2 = 25^2$ → Substitution
$x^2 + 400 = 625$ → Simplify
$x^2 = 225$ → Subtract 400 from both sides
$\sqrt{x^2} = \sqrt{225}$ → Square root both sides
$x = 15$ → Simplify

Answer: $BE = 15$

Ex 4) Given Square RSTU where $SU = 6$ and $RT = 8$. What is the length of ST, the perimeter and area of Square RSTU?



Work:

*SU & RT are whole segments. To find ST we need segments SV and VT. So we will divide SU and RT by 2.

SV=3 and TV=4

To find ST, use Pythagorean Theorem:
$a^2 + b^2 = c^2$
$3^2 + 4^2 = x^2$ → Substitution
$25 = x^2$ → Simplify
$\sqrt{25} = \sqrt{x^2}$ → Square root both sides
$5 = x$ → Simplify

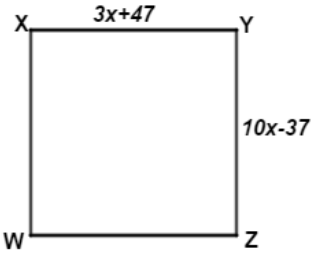
Answer: $ST = 5$
Perimeter: $5 + 5 + 5 + 5 = 20$
Area: (5)(5)=25

Ex 5) Given Square WXYZ. Find x.

Work:

*All sides are congruent

$10x - 37 = 3x + 47$
$7x - 37 = 47$ → Subtraction of 3x on both sides
$7x = 84$ → Add 37 to both sides
$x = 12$ → Divide both sides by 7



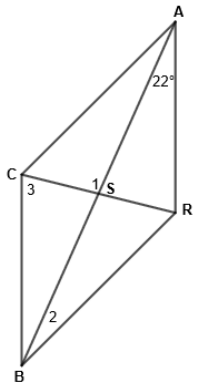
Directions: Use your knowledge of the properties of rhombi and squares to solve each of the following.

1) Find the values of the missing angles in Rhombus CARB.

$m\angle 1 =$_____

$m\angle 2 =$_____

$m\angle 3 =$_____

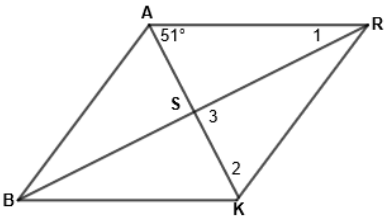


2) Find the values of the missing angles in Rhombus BARK.

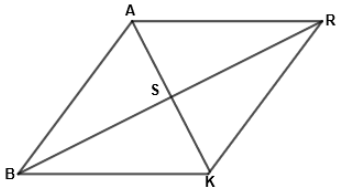
$m\angle 1 =$_____

$m\angle 2 =$_____

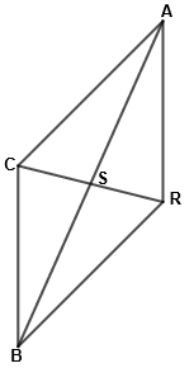
$m\angle 3 =$_____



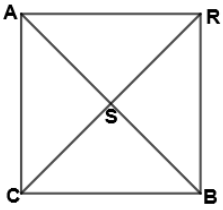
3) Given Rhombus BARK. The $m\angle SAR = (27x - 12)°$ and the $m\angle SAB = (20x + 9)°$. Find the value of x and the $m\angle SAB$?



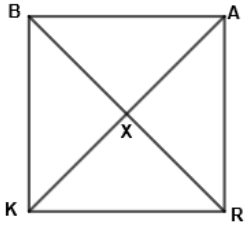
4) Given Rhombus CARB, if $m\angle SBC = (9x - 43)°$ and $m\angle SBR = (3x + 29)°$, what is the value of x?



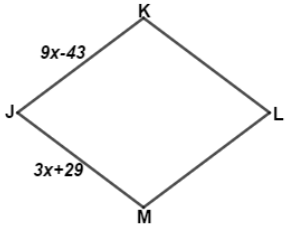
5) Given Square CARB with $AB = 32$ and $CR = 60$. What is the length of CB and the perimeter of Square CARB?



6) Given Square BARK, if BA=37 and BX=12. What is the length of AX and the area?



7) Find the area and perimeter of Rhombus JKLM.



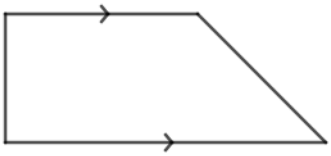
8) Square GHIJ has side $GH = 15x - 19$ and $JG = 11x - 3$. What is the value of x? (Hint: Draw a picture.)

Day 4- Trapezoids & Isosceles Trapezoids

Trapezoid:

Properties:

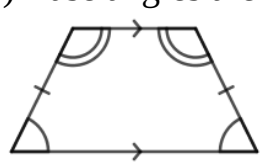
1) One pair of parallel sides
2) Consecutive interior (same side interior) angles are supplementary

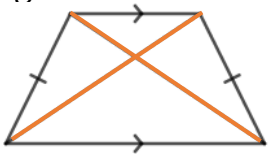


Isosceles Trapezoid

Properties: All properties of Trapezoids plus:

1) Diagonals are congruent
2) Base angles are congruent





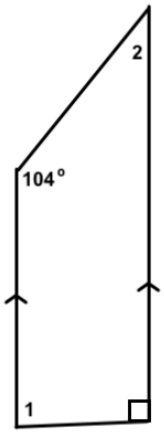
Ex 1) Given the following trapezoid, find the two missing angles.

Work:

* Consecutive Interior angles are supplementary.

$m\angle 1 = 90° \rightarrow 180 - 90 = 90$

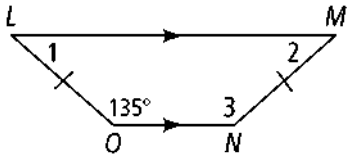
$m\angle 2 = 76° \rightarrow 180 - 104 = 76$



Ex 2) Given Isosceles Trapezoid LMNO. Find the measures of the missing angles.

*Base angles are congruent.

So: $\angle L \cong \angle M$ and $\angle O \cong \angle N$



Work:

$m\angle 1 = 45° \rightarrow 180 - 135 = 45$. Consecutive Interior angles are supplementary.

$m\angle 2 = 45° \rightarrow$ Base angles are congruent

$m\angle 3 = 135° \rightarrow$ Base angles are congruent.

Ex 3) Given trapezoid MNOP with base $\overline{MN}$. If $m\angle M = (7x + 36)°$ and $m\angle P = (3x + 4)°$, what is the $m\angle M$?

*Consecutive Interior angles are supplementary,

Work:

$7x + 36 + 3x + 4 = 180$

$10x + 40 = 180 \rightarrow$ Simplify

$10x = 140 \rightarrow$ Subtract 40 from both sides

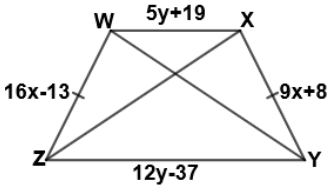
$x = 14 \rightarrow$ Divide by 10 on both sides.

$m\angle M = 7(14) + 36 = 134 \rightarrow$ Substitute in for angle M

Answer: $m\angle M = 134°$

Ex 4) Given Isosceles Trapezoid WXYZ. <u>Set up</u> the equations to solve for x and y.

* Isosceles Trapezoids have one pair of congruent sides and diagonals are congruent.



Answers:

Equation for x: $16x - 13 = 9x + 8$

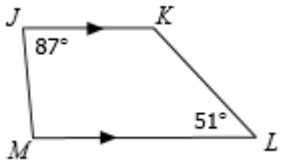
Equation for y: $12y - 37 = 5y + 19$

Directions: Use your knowledge of the properties of trapezoids and isosceles trapezoids to solve each of the following.

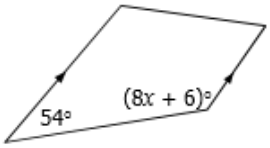
1) Given Trapezoid JKLM, find the $m\angle M$ and $m\angle K$.

$m\angle M =$_____

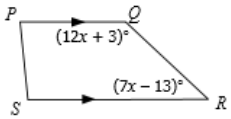
$m\angle K =$_____



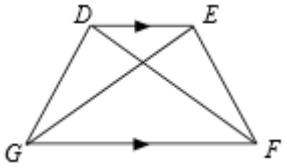
2) Solve for x in the trapezoid below.



3) What is the $m\angle R$ in trapezoid PQRS?



4) DEFG is an Isosceles Trapezoid.



$\overline{DG} \cong$_____

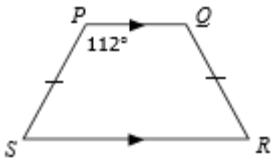
$\overline{DF} \cong$_____

5) PQRS is an isosceles trapezoid. Find the other angles.

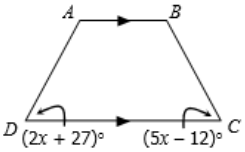
$m\angle Q =$_____

$m\angle R =$_____

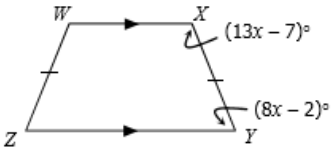
$m\angle S =$_____



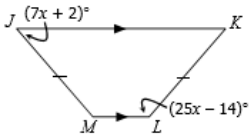
6) If ABCD is an isosceles trapezoid. What is the value of x?



7) Solve for x.



8) Jennifer had the following question on her quiz: Solve for x.



She used the following equation to solve for x:

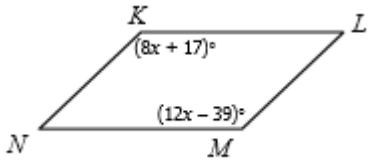
$$7x + 2 = 25x - 14$$

Was Jennifer correct or incorrect? Explain your reasoning.

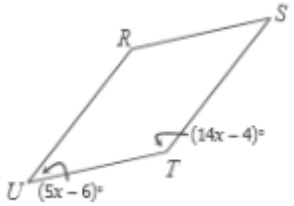
Day 5- Review of Parallelograms, Rectangles, Rhombi, Squares, Trapezoids & Isosceles Trapezoids.

Directions: Use your knowledge of Parallelograms, Rectangles, Rhombi, Squares, Trapezoids & Isosceles Trapezoids to solve the following questions.

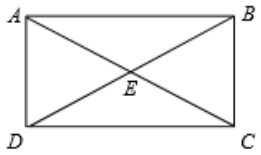
1) Find the $m\angle N$ in Parallelogram KLMN



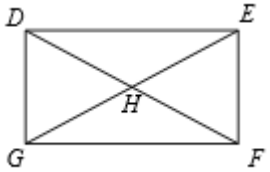
2) Parallelogram RSTU, what is the $m\angle R$?



3) Given Rectangle ABCD, if AC= 30 and AD=18, What is DC and the perimeter of the rectangle?



4) Given Rectangle DEFG. if $m\angle EDH = (4x - 5)°$ and $m\angle HDG = (6x + 35)°$. What is the value o x?

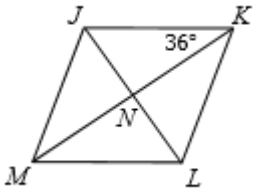


5) Given Rhombus JKLM, find the measure of the following angles:

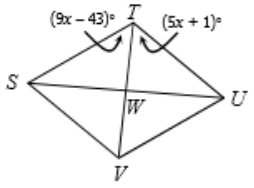
$m\angle NML =$_____

$m\angle JNM =$_____

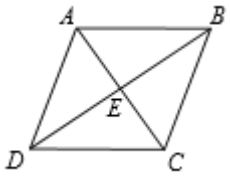
$m\angle NJK =$_____



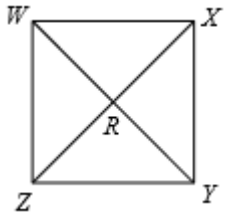
6) If STUV is a rhombus, find $m\angle SVU$.



7) If ABCD is a rhombus and $AD = 4x + 2$ and $DC = 7x - 13$. What is the value of x and the perimeter?



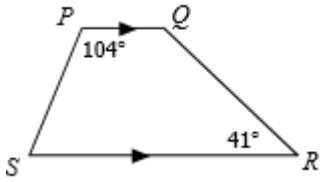
8) Quadrilateral WXYZ is a square. If $WX = 9x + 1$ and $XY = 13x - 11$, what is the value of x and the area of the square?



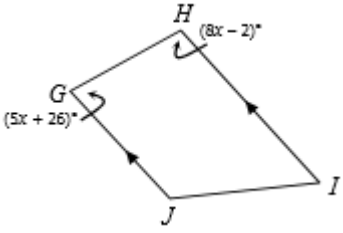
9) Given Trapezoid PQRS, find the missing angles.

$m\angle Q =$_____

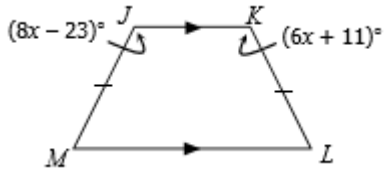
$m\angle S =$_____



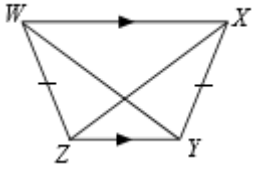
10) Solve for x in Trapezoid GHIJ.



11) Solve for x.



12) If $WY = 15x - 2$ and $XZ = 9x + 10$. What is length of WY?

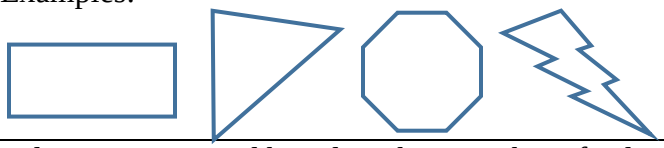


Week 3 **Polygons Day 1**

Notes:

A **polygon** is a shape with at least 3 straight sides. See examples and non-examples below:

Examples:



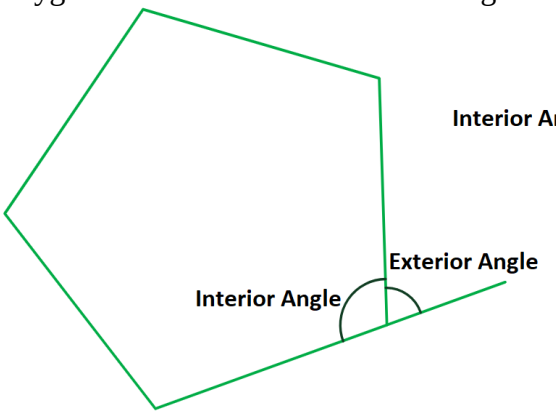
Non-examples:

Polygons are named based on their number of sides.

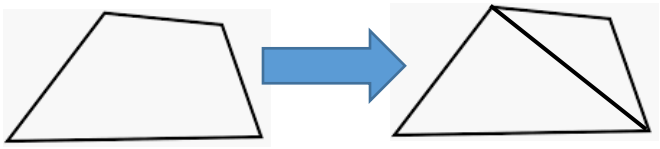
- 3 sides is a triangle
- 4 sides is a quadrilateral
- 5 sides is a pentagon
- 6 sides is a hexagon
- 7 sides is a heptagon
- 8 sides is an octagon
- 9 sides is a nonagon
- 10 sides is a decagon
- 12 sides is a dodecagon

All other polygons are simply named 11-gon meaning eleven sides, 13-gon meaning thirteen sides, 20-gon meaning twenty sides, etc.

Polygons have interior and exterior angles:



Recall that a three-sided polygon, aka a triangle, has an interior sum of 180° (meaning all three angles add up to equal 180). Using that understanding, imagine that a quadrilateral is divided into two triangles by drawing a diagonal:



Since a quadrilateral can be represented as two Triangles, there are two sets of 180:

180 + 180 = 360

This is true of all quadrilaterals, therefore all quadrilaterals have an interior sum of 360

There is a relationship between the number of sides, represented by *n*, of a polygon and the number of triangles that can be drawn by non-intersecting diagonals:

3 sides means 1 triangle exists,

4 sides means 2 triangles exist (as we saw in the previous example),

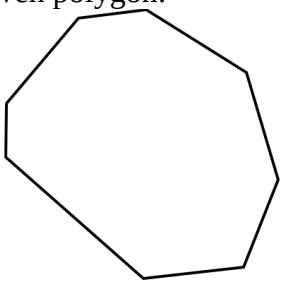
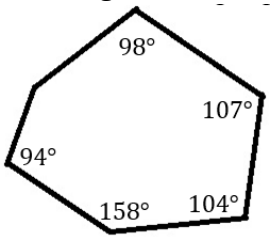
5 sides means 3 triangles exist

This pattern continues to reveal that the number of triangles can be represented by "$n-2$" (again, where n is the number of sides of a polygon.

Therefore, the **sum of the interior angles** of any polygon can be found using the following formula:

$(\boldsymbol{n-2})\cdot \mathbf{180}$ where n is the number of sides of the polygon

See solutions to some examples on the next page:

Ex1: Find the sum of the interior angles of the given polygon:	Ex2: Find the sum of the interior angles of a dodecagon:	Ex3: Find the missing angle in the hexagon
 Solution: $n = 8$ $(8-2)\cdot 180 =$ **1080**	Solution: $n = 12$ $(12-2)\cdot 180 =$ **1800**	Solution: $n = 6$ $(6-2)\cdot 180 = 720$ Using algebra, we refer to the missing angle as x:  $x + 98 + 107 + 104 + 158 + 94 = 720$ Combining like terms: $x + 561 = 720$ Subtracting 561 from both sides yields the answer: $x = 159$
Final note: **The sum of the exterior angles of *any* polygon is always 360°**		

Now you try:

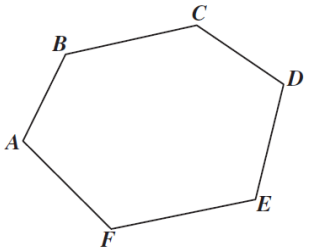
Sum of Interior Angles: $(n-2)\cdot 180$ **Sum of Exterior Angles: 360**

Ex1) Find the sum of the interior angles of a dodecagon: _____

Ex2) Find the sum of the exterior angles of a nonagon: _____

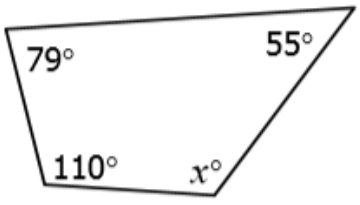
Ex3) Find the sum of the interior angles of a 15-gon: _____

Ex4)

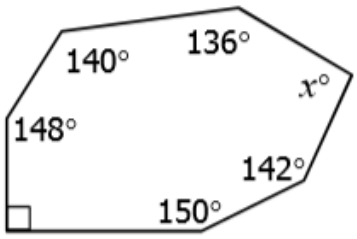


Given the polygon shown above,
$m\angle A + m\angle F + m\angle E + m\angle D + m\angle C + m\angle B =$ ____

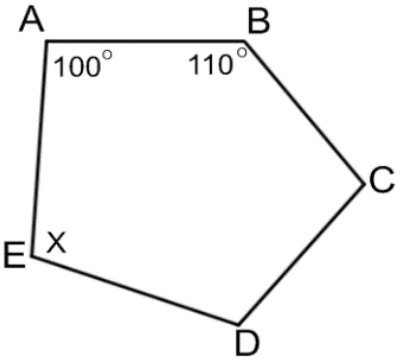
Ex5) Find the value of x



Ex6) Solve for x



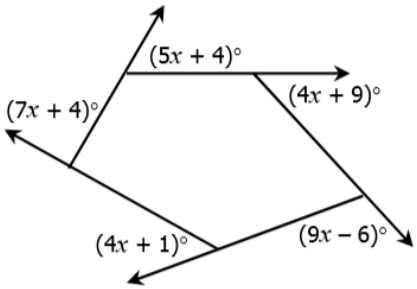
Ex7) In Pentagon ABCDE, angle A and angle C are congruent as well angle B being congruent to angle D. Solve for x.



Polygons Day 2

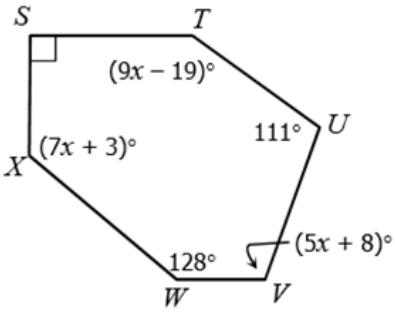
Notes: Using algebra and the Interior/Exterior Sums from Day 1, you can solve problems like these:

Ex1: Find the value of x:



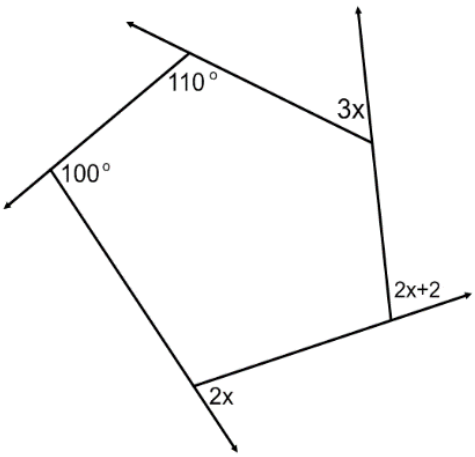
STEPS	
1. Interior or Exterior?	Exterior (the variable is outside of the polygon)
2. Find total.	Total exterior sum is always = 360
3. Set up equation.	$7x + 4 + 5x + 4 + 4x + 9 + 9x - 6 + 4x + 1 = 360$
4. Solve equation.	Combine like terms $29x + 12 = 360$ Subtract 12 on both sides $29x = 348$ Divide both sides by 29 $x = 12$
5. Plug back in?	This final step is to check that you have solved for what the problem has asked. In this example, we were instructed to just find x, which we have done, so the final answer is 12.

Ex2: Find $m\angle V$



STEPS	
1. Int or Ext?	Interior (the variable is inside of the polygon)
2. Find total.	Total interior sum: $(6-2)\cdot 180 = 720$
3. Set up equation.	$90 + 9x - 19 + 111 + 5x + 8 + 128 + 7x + 3 = 720$
4. Solve equation.	Combine like terms: $21x + 321 = 720$ Subtract 321 on both sides: $21x = 399$ Divide both sides by 21 $x = 19$
5. Plug back in?	Because this problem asked for the measure of angle *V*, 19 is *not* the final answer. You must now plug in the value of x into the expression for angle *V*: $m\angle V = 5 \cdot 19 + 8$ $m\angle V = 95 + 8$ $m\angle V = 103$ The final answer is 103°

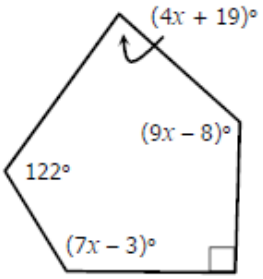
Ex3: Find the value of x.



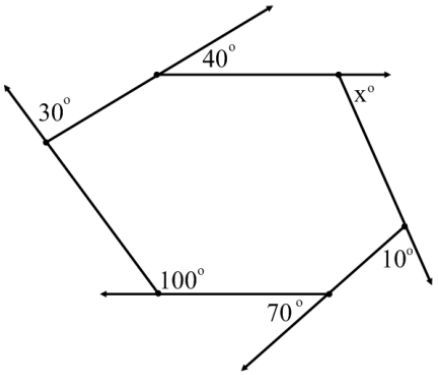
STEPS	
1. Int or Ext?	Exterior (the variable is outside of the polygon)
2. Find total.	Total exterior sum is always = 360
3. Set up equation.	*Note that two of the given angles are interior instead of exterior. Recall that an interior angle is supplementary to its exterior to find that the two missing exterior angles are 80° and 70° $3x + 2x + 2 + 2x + \mathbf{80} + \mathbf{70} = 360$
4. Solve equation.	Combine like terms $7x + 152 = 360$ Subtract 152 on both sides $7x = 208$ Divide both sides by 7 $x = 29.7$
5. Plug back in?	In this example, we were asked to just find x, which we have done, so you do *not* need to plug in. The final answer is 29.7

Now you try:

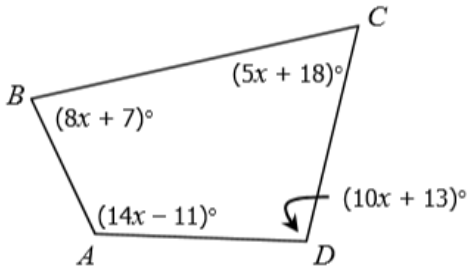
1. Find the value of x.



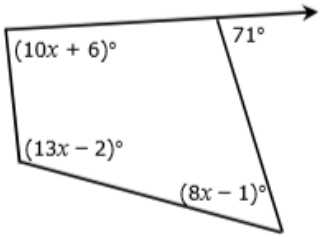
2. Solve for x



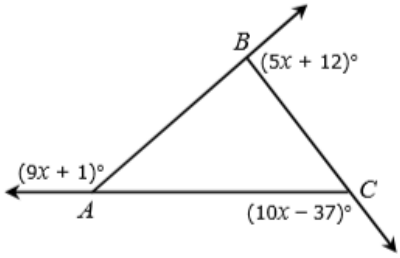
3. Find $m\angle BCD$



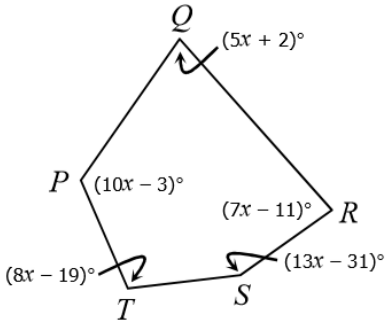
4. Find the value of x



5. Determine the measure of $\angle ABC$



6. Find $m\angle PTS$

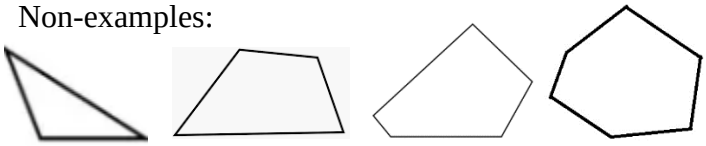


Polygons Day 3

Some polygons are **regular** meaning all sides are of equal length, and all angles are of equal measure.

Examples:

Non-examples:



Consider the regular hexagon to the right. You already know that to find the interior sum, use the formula: $(n-2)\cdot 180$ → $(6-2)\cdot 180 = 720$. Given that the hexagon is regular, you also know that all six angles are equal to each other. So, divide the total interior sum, 720, by the number of angles/sides, 6: $720 \div 6 = 120°$

This is how you can find any interior angle of any polygon, *as long as it is regular.*

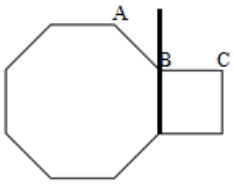
Formula for one interior angle of a *regular* polygon: $\frac{(n-2)\cdot 180}{n}$ Formula for one exterior angle of a *regular* polygon: $\frac{(n-2)\cdot 180}{n}$

Similarly, to find one *exterior* angle of a regular polygon, divide the total *exterior* sum by the number of angles/sides:

Formula for one exterior angle of a *regular* polygon: $\frac{360}{n}$

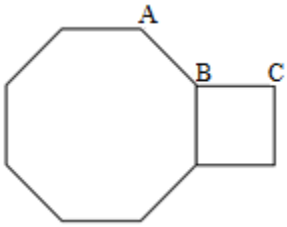
Reminder: An interior angle is supplementary to its exterior. Therefore, if you know an interior angle, you can find its exterior simply by subtracting the interior from 180 (and vice versa). For instance, in the example above in which we found the interior angle of a regular hexagon is 120°, you could then find the exterior angle by using the formula $\frac{360}{n}$ or you could subtract 120 from 180. Both are acceptable methods to find an exterior angle, and both methods will give the answer of 60°.

Sometimes, you may be asked to find an exterior angle formed by two shapes, as in the example below. In this case, imagine the line representing the shared side is extended, like this: Now, it is easier to see that angle *ABC* is composed of two angles, one exterior angle of the octagon and one exterior angle of the square. So, simply use the formula $\frac{360}{n}$ to find each exterior angle and add them together.



Follow the steps below.

Ex) Find the $m\angle ABC$ if the square and octagon are regular polygons.

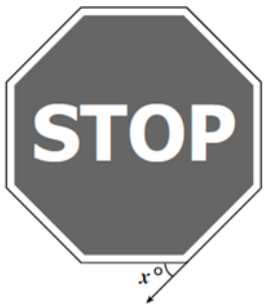


STEPS	
1. Interior or Exterior?	Exterior
2. What is the measure of 1 exterior angle of the first polygon?	$\frac{360}{8} = 45°$
3. What is the measure of 1 exterior angle of the second polygon?	$\frac{360}{4} = 90°$
4. Add the two exterior angles together to find the total angle.	$45° + 90° = 135°$

Now you try:

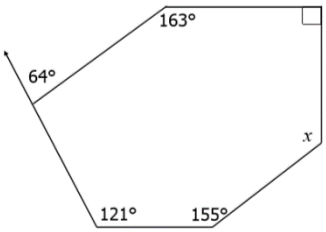
1. Consider the two formulas: $\frac{(n-2)\cdot 180}{n}$ and $\frac{360}{n}$. Circle all of the following problems can be solved using one of the two formulas:

A.



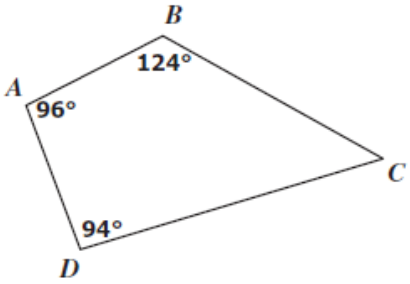
B.

Find the value of x in the figure below:



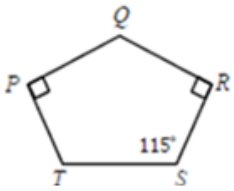
C.

What is the measure of $\angle C$ in quadrilateral $ABCD$?

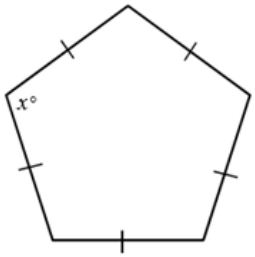


D.

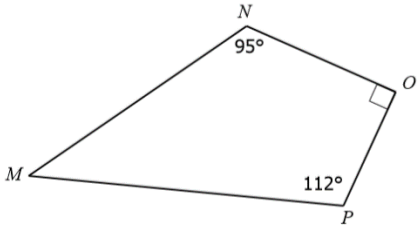
Given the polygon below, if $\angle T \cong \angle S$, find $m\angle Q$.



E.



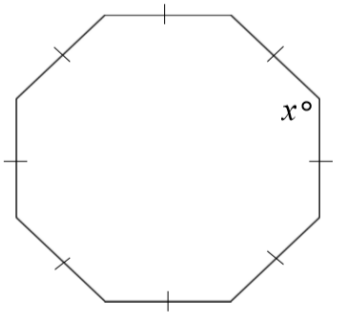
F. What is the measure of $\angle NMP$?



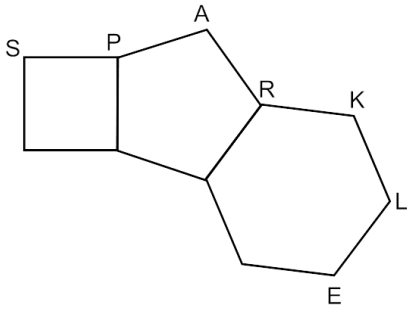
2. Find the measure of one interior angle of a regular decagon.

3. Determine the measure of one exterior angle of a regular 30-gon.

4. Find the value of x



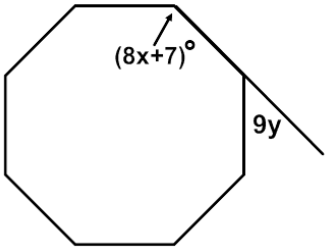
5. **Determine m∠ SPA**



6. Find the measure of one of the exterior angles of the given polygon (assume the polygon is regular)

7. Given the image below, which of the following equations is correct? (assume the polygon is regular)

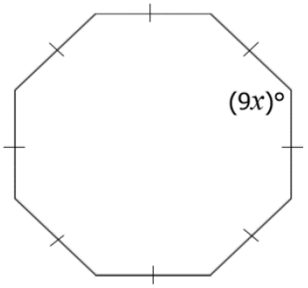
A. $8x + 7 = 45$
B. $8x + 7 = 35$
C. $9y = 45$
D. $9y = 135$



Polygons Day 4

Notes: Using algebra and the Interior/Exterior Angle Formulas from Day 3, you can solve problems like these:

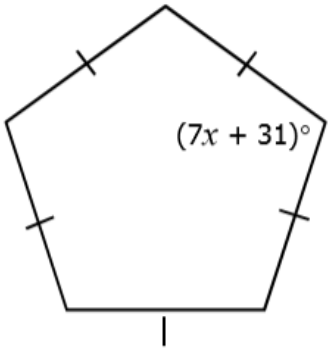
Ex. Solve for x



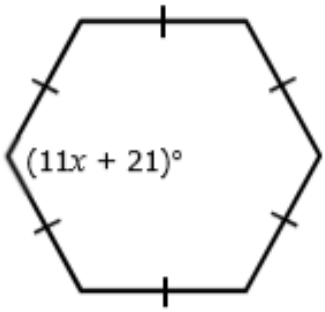
STEPS	
1. Interior or Exterior?	Interior
2. Find one angle measure	$\frac{(n-2)\cdot 180}{n} \rightarrow \frac{(8-2)\cdot 180}{8} = 135°$
3. Set up equation.	$9x = 135$
4. Solve equation.	$9x = 135$ Divide both sides by 9 $x = 15$

Now you try:

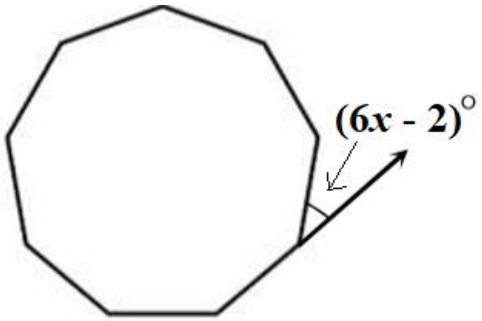
1. Find the value of x:



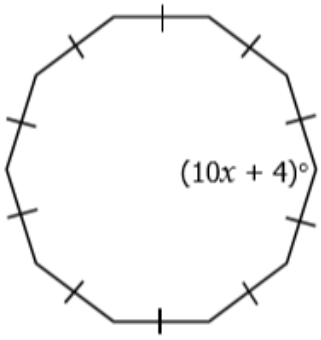
2. Solve for x:



3. Find x



4. Determine the value of x



5. The interior angle measure of a regular dodecagon is represented by the expression $(3a - 24)°$. Find the value of a.

6. The exterior angle measure of an equilateral triangle is represented by the expression $(5p + 45)°$. Find the value of p.

Polygons Day 5

Last Day on Polygons! Yaaayyyy!

There are some cases in which you can be asked to find the number of sides. We can look at the formula for one exterior angle, $\boldsymbol{e = \frac{360}{n}}$, where *e* is the exterior angle and *n* is still the number of sides. By solving this formula for *n*, we get a formula for the number of sides of a regular polygon: $\boldsymbol{n = \frac{360}{e}}$

Ex. Find the number of sides of a polygon that has an exterior angle measure of 18°.

Solution: Plug in the exterior angle measure to the formula $\boldsymbol{n = \frac{360}{e}}$ → $\boldsymbol{n = \frac{360}{18} = 20}$
Final Answer: 20 sides.

Ex. What is the name of a polygon that has an interior angle measure of 140°?

Solution: We cannot plug in an exterior angle measure because we don't have one, yet! Recall that an interior angle is supplementary to its exterior. Therefore, if the interior angle measures 140°, then the exterior angle must be 40° (because 180 – 140 = 40). Now, we can plug in the exterior angle to the formula $\boldsymbol{n = \frac{360}{e}}$ → $\boldsymbol{n = \frac{360}{40} = 9}$
Final Answer: The name of a 9-sided shape is a **nonagon**.

Now you try:

1. How many sides does a regular polygon have if its exterior angle measures 15°?

2. How many sides does a regular polygon have if its exterior angle measures 6°?

3. How many sides does a regular polygon have if its interior angle measures 160°?

4. What is the name of a regular polygon that has an interior angle measuring 150°?

5. Imagine that the corner of a regular polygon is ripped off as shown below. How many sides does it have?